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Problem Set 5

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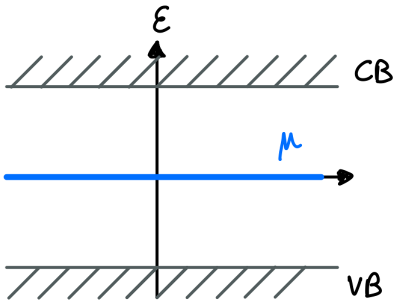
Exercise 1 : Abrupt p-n junction

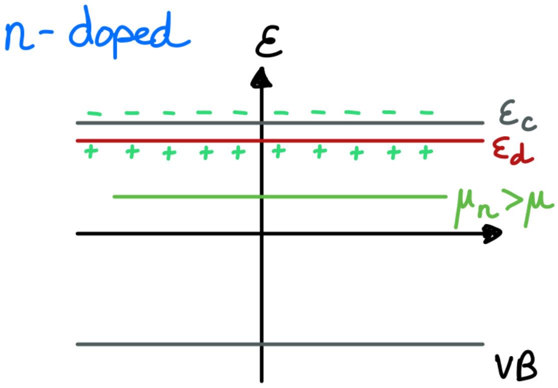
We study a diode formed by an abrupt junction between n-doped silicon (on the right) with a density $N_\mathrm{a} = 2 \times 10^{16}$ cm^{-3} of donors and p-doped silicon (on the left) with an acceptor density $V_\mathrm{a} = 5 \times 10^{15}$ cm^{-3}. Assume room temperature ($T = 300K$) and use the following parameters for silicon : intrinsic carrier density $n_\mathrm{i} = 1.08 \times 10^{10}$ cm^{-3}, energy gap $E_\mathrm{g} = 1.12$eV ; relative permittivity (dielectric constant) $\epsilon_r = 11.7$.

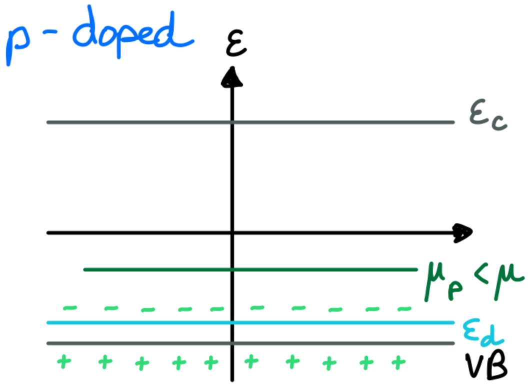
a) Sketch the energy band diagram for pure silicon, n-doped silicon, and p-doped silicon (in the non-degenerate case).

pure silicon :



doped silicon :





b) Explain why pure silicon is an insulator with very low conductivity at $T = 300$K. Explain why the conductivity is strongly increased when silicon is doped n or p.

$E_g \gg k_B T$ so it is an insulator at room temperature. The conductivity goes up when we dope. Indeed, the new gap, *i.e.* the difference in energy in-between either ϵ_a and ϵ_v or ϵ_c and ϵ_d is much smaller than the initial gap : $E_g' \ll E_g$, and thus $E_g' \sim k_B T$, so it allows to promote electron and thus increasing the conductivity.

c) Assume that all dopants are ionized and calculate the chemical potentials with respect to the conduction or valence band edges in n-doped and p-doped silicon before the junction is formed.

$$\begin{cases} n_c(T) = N_c(T)e^{(\mu-\epsilon_c)/k_BT} \\ p_v(T) = P_v(T)e^{(\epsilon_v-\mu)/k_BT} \end{cases}$$

and we know that

$$n_i(T) = \sqrt{n_c(T)p_v(T)}$$

For a n-doped, $n_c = N_d$, and thus $p_v = n_i^2/N_d$, doing the same calculation, we get for p-doped $p_v = N_a$ and $n_c = n_i^2/N_a$. To extrct the chemical potential we can now do the quotient,

$$\begin{aligned} \frac{n_c(T)}{p_v(T)} &= \frac{N_c(T)}{P_v(T)}e^{2\beta\mu}e^{-\beta(\epsilon_c+\epsilon_v)} \\ &= \left(\frac{m_e^*}{m_h^*}\right)^{3/2} e^{2\beta\mu}e^{-\beta(\epsilon_c+\epsilon_v)} \end{aligned}$$

Thus we get

$$\mu = \frac{\epsilon_c+\epsilon_v}{2} + \frac{k_BT}{2}\left\{\ln\left(\frac{n_c(T)}{p_v(T)}\right) + \frac{3}{2}\ln\left(\frac{m_h^*}{m_e^*}\right)\right\}$$

So, in the n-doped case we get $n_c/p_v = (N_d/n_i)^2$,

$$\mu = \frac{\epsilon_c+\epsilon_v}{2} + k_BT\ln\left(\frac{N_d}{n_i}\right) + \frac{3}{4}k_BT\ln\left(\frac{m_h^*}{m_e^*}\right)$$

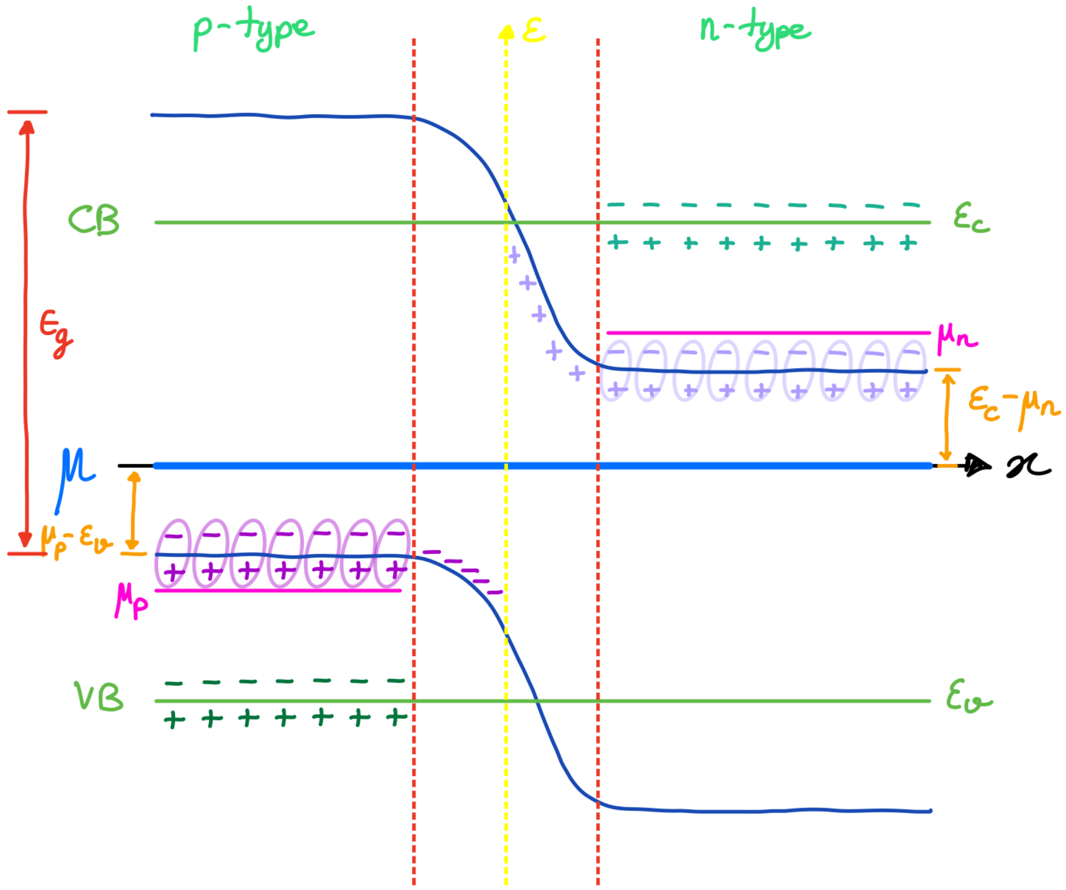
and in the p-doped case we get $n_c/p_v = (n_i/N_a)^2$,

$$\mu = \frac{\epsilon_c+\epsilon_v}{2} - k_BT\ln\left(\frac{N_a}{n_i}\right) + \frac{3}{4}k_BT\ln\left(\frac{m_h^*}{m_e^*}\right)$$

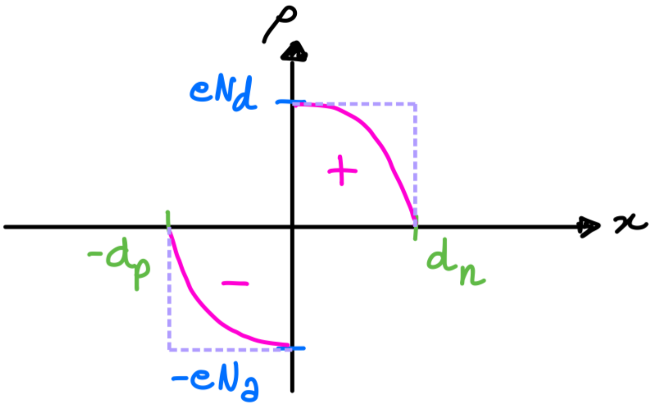
If we do the numerical application one can get,

$$\begin{cases} \mu_n - \epsilon_c = -0.16 \text{ eV} \\ \mu_p - \epsilon_v = +0.19 \text{ eV} \end{cases}$$

d) Sketch the energy band diagram of the junction at equilibrium. What happens at the interface? Deduce the profile of the charge density in the $p - n$ junction.



from that we can deduce the charge density,



e) A potential step appears at the interface of the p-n junction (also called diffusion barrier). Express the diffusion potential V_b that defines the barrier height in equilibrium eV_b in terms of n_i, N_a and N_d, and calculate its value.

We can write

$$eV_b = E_g - (\epsilon_c - \mu_n) - (\mu_p - \epsilon_v) = \mu_n - \mu_p = 0.69\text{ eV}$$

We can also write,

$$\mu_n - \mu_p = k_B T\left[\ln\left(\frac{N_d}{n_i}\right) + \ln\left(\frac{N_a}{n_i}\right)\right] = k_B T\ln\left(\frac{N_d N_a}{n_i^2}\right)$$

f) Determine the spatial dependence of the electric field in the junction region.

From electromagnetism we know that

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon} \qquad\qquad \vec{E} = -\vec{\nabla}\phi$$

And so we get Poisson's equation

$$\Delta\phi = -\frac{\rho}{\epsilon}$$

ρ is independent of y and z, so we can calculate only the x contribution.

$$\rho(x) = \begin{cases} 0 & x < -d_p \\ -eN_a & -d_p < x < 0 \\ eN_d & 0 < x < d_n \\ 0 & x > d_n \end{cases}$$

So we can find the electric field,

$$E_x(x) = \begin{cases} c_1 & x < -d_p \\ -eN_a(x + c_2) & -d_p < x < 0 \\ eN_d(x + c_3) & 0 < x < d_n \\ c_4 & x > d_n \end{cases}$$

Using the continuity of the electric field at its boundary, one get :

$$\frac{c_2}{c_3} = -\frac{N_d}{N_a}$$

We also know that $E_x(\pm\infty) = 0$ so we get

$$c_1 = 0 \qquad c_2 = d_p \qquad c_3 = -d_n \qquad c_4 = 0$$

So we have

$$E_x(x) = \begin{cases} 0 & x < -d_p \\ -eN_a(x + d_p) & -d_p < x < 0 \\ eN_d(x - d_n) & 0 < x < d_n \\ 0 & x > d_n \end{cases}$$

g) We choose the origin $x = 0$ of the x-axis (orthogonal to the junction plane) at the p-n interface and denote by x_{n} and x_{p} the outer edges of the charge depletion zones on the n-side and the p-side of the junction, respectively. Show that there is a relation connecting $x_{\mathrm{n}}, x_{\mathrm{p}}, N_{\mathrm{a}}$ and N_{d}.

$$N_d d_n = N_a d_p$$

h) Let V_{n} and V_{p}, denote the value of the electrostatic potential outside the depletion zones on the n-side and the p-side of the junction, respectively. Determine the position dependence of the electrostatic potential through the junction Use the expression found for the potential step V_{b}, to express the values of x_{n} and x_{p} as a function of n_{i}, N_{a} and N_{d}. Deduce an expression for the width w of the charge depletion zone. Calculate the numeric values of x_{n}, x_{p}, and w.

$$\phi(x) = \begin{cases} V_p & x < -d_p \\ +\dfrac{eN_a}{2\epsilon}(x+d_p)^2 + c_2 & -d_p < x < 0 \\ -\dfrac{eN_d}{2\epsilon}(x-d_n)^2 + c_3 & 0 < x < d_n \\ V_n & x > d_n \end{cases}$$

Once again using continuity we get

$$c_2 = V_p \qquad\qquad c_3 = V_n$$

And using the continuity in 0 we get

$$d_n = \sqrt{\frac{2\epsilon(V_p - V_n)}{e}\frac{N_d}{(N_a+N_d)N_a}} \qquad\qquad d_p = \sqrt{\frac{2\epsilon(V_p - V_n)}{e}\frac{N_a}{(N_a+N_d)N_d}}$$

$$\begin{cases} d_p = 374 \text{ nm} \\ d_n = 94 \text{ nm} \end{cases} \longrightarrow w = 468 \text{ nm}$$

i) Show that minority carrier densities outside the depletion zone are exponentially dependent on V_{b}.

$$\begin{cases} n_{c,p}(T) = N_c(T)e^{-\beta(\epsilon_c - \mu - e\phi(x))} = N_c(T)e^{-\beta(\epsilon_c - \mu - eV_p)} \\ n_{c,n}(T) = N_c(T)e^{-\beta(\epsilon_c - \mu - e\phi(x))} = N_c(T)e^{-\beta(\epsilon_c - \mu - eV_n)} \end{cases}$$

Thus,

$$n_{c,p}(T) = n_{c,n}(T)e^{-eV_b\beta}$$

j) A positive voltage is applied to the p-side of the junction while the n-side remains grounded. V_j is noted the potential difference created by the external voltage source at the junction. We consider the case where $V_\mathrm{j} < V_\mathrm{b}$.

- Represent the energy band diagram of the junction submitted to V_j.

- What is the value of the new diffusion potential V_b' ? Express it as a function of V_b and V_j.

$$V_b' = V_b - V_j$$

- What is the new width of the charge depletion zone w' ? Express it as a function of w, V_b, and V_j.

$$w' \propto \sqrt{V_b'} = w\sqrt{1 - \frac{V_j}{V_b}}$$

- What are the new minority carrier densities n_p' (electron density at the p-side) and p_n' (hole density at the n-side) at both ends of the charge depletion zone ? Express them as functions of n_p, p_n and V_j.

$$n_{c,p}' = n_{c,p}e^{e\beta V_j}$$

- Deduce the existence of a diffusion current in the junction.

  Due to the fact that there is a density gradient.

k) It is assumed that the hole density on the n-doped side for $x > x_\mathrm{n}'$ decreases as $p(x) = p_\mathrm{n} + (p_\mathrm{n}' - p_\mathrm{n}) \exp\left[-(x - x_\mathrm{n}')/L_\mathrm{p}\right]$ to reach asymptotically the hole density p_n on the n-side far from the interface, where L_p is the hole diffusion length. Show that the associated hole diffusion current density at $x = x_\mathrm{n}'$ is

$$J_{\mathrm{diff,p}}(x_\mathrm{n}') = -eD_\mathrm{p}\left[\frac{\mathrm{d}p_\mathrm{n}(x)}{\mathrm{d}x}\right]_{x=x_\mathrm{n}'} = \frac{eD_\mathrm{p}p_\mathrm{n}}{L_\mathrm{p}}\left(\exp\left[\frac{eV_\mathrm{J}}{k_\mathrm{B}T}\right] - 1\right)$$

where D_p is the hole diffusion constant. Neglect generation and recombination processes. Explain why the total hole current is approximately given by $J_{\mathrm{diff,p}}(x_\mathrm{n}')$ and use a similar reasoning for the electron current to show that the total current reads

$$I_{\mathrm{F}} = I_{\mathrm{S}}\left(\exp\left[\frac{eV_{\mathrm{J}}}{k_{\mathrm{B}}T}\right] - 1\right) \qquad \text{with} \qquad I_{\mathrm{S}} = Sen_{\mathrm{i}}^2\left(\frac{D_{\mathrm{P}}}{L_{\mathrm{p}}N_{\mathrm{d}}} + \frac{D_{\mathrm{n}}}{L_{\mathrm{n}}N_{\mathrm{a}}}\right)$$

S being the cross-section of the device.

$$p(x) = p_n + (p_n' - p_n)e^{-(x-x_n')/L_p}$$

$$\begin{aligned} J_{\text{dif},p}(x_n') &\equiv -eD_p\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)\Bigg|_{x=x_n'} \\ &= -eD_p(p_n' - p_n) \times \left(-\frac{1}{L_p}\right) \\ &= \frac{eD_p(p_n' - p_n)}{L_p} \\ &= \frac{eD_p p_n}{L_p}\left(e^{eV_J/k_BT} - 1\right) \end{aligned}$$

$$n(x) = n_p + (n_p' - n_p)e^{(x-x_p')/L_p}$$

$$J_{\text{dif},n}(x_p') = -\frac{eD_n n_p}{L_n}\left(e^{eV_J/k_BT} - 1\right)$$

And so,

$$\begin{aligned} I_F &= S\left(J_{\text{dif},p}(x_n') + J_{\text{dif},n}(x_p')\right) \\ &= Se\left(e^{eV_J/k_BT} - 1\right)\left(\frac{D_n n_p}{L_n} + \frac{D_p p_n}{L_p}\right) \\ &= Sen_i^2\left(e^{eV_J/k_BT} - 1\right)\left(\frac{D_n}{L_nN_a} + \frac{D_p}{L_pN_d}\right) \\ &= I_S\left(e^{eV_J/k_BT} - 1\right) \end{aligned}$$

Diode rectifying bias.

l) Show that the current at strong reverse bias is limited by $I_{\mathrm{R}} = -I_{\mathrm{S}}$.

$$\lim_{V_J \to -\infty} I_F = -I_S$$